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Quasi-degenerate neutrinos and lepton flavor violation in supersymmetric models

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Abstract. In supersymmetric models the misalignment between fermion and sfermion families introduces unsuppressed flavor-changing processes. Even if the mass parameters are chosen to give no flavor violation, family dependent radiative corrections make this adjustment not stable. We analyze the rate of $\ell \to \ell' \gamma$ in SUSY-GUT models with three quasi-degenerate neutrinos and universal scalar masses at the Planck scale. We pay special attention to a recently proposed scenario where the low-energy neutrino mixings are generated from identical quark and lepton mixings at large scales. We show the following. (i) To take universal slepton masses at the GUT scale is a very poor approximation, even in *no-scale* models. (ii) For large neutrino Yukawa couplings the decay $\mu \to e\gamma$ would be observed in the planned experiment at PSI. (iii) For large values of $\tan\beta$ the tau coupling gives important corrections, pushing $\mu \to e\gamma$ and $\tau \to \mu\gamma$ to accessible rates. In particular, the non-observation of these processes in the near future would exclude the scenario with unification of quark and lepton mixing angles. (iv) The absence of lepton flavor violating decays in upcoming experiments would imply a low value of $\tan\beta$, small neutrino couplings, and large ($\gtrsim 250\,\mathrm{GeV}$) SUSY-breaking masses.

1 Introduction

Supersymmetric (SUSY) extensions of the standard model introduce new sources of flavor non-conservation. Essentially, the three fermion families get SUSY masses from Yukawa interactions, whereas their scalar partners must get additional SUSY-breaking masses with a different origin. In general this will produce a misalignment between fermions and sfermions and then tree level flavor changing couplings to gauginos and higgsinos [1].

An acceptable SUSY-breaking mass matrix $\mathbf{m}^2$ should be (nearly) diagonal after the rotation diagonalizing the corresponding fermion matrix. The most economical solution is that $\mathbf{m}^2 \propto \mathbf{1}$, and the three scalar masses coincide at the tree level. In particular, in the most popular scenario SUSY is broken in a hidden sector only connected via gravitational interactions with the standard model. Being universal, gravity could generate identical masses for the three sfermion families near the Planck scale [2]. Obviously, this universality would be broken by radiative corrections [3,4], as the observed pattern of fermion masses tells us that the interactions of the three families with the Higgs fields are very different.

The renormalization-group (RG) corrections to universal SUSY-breaking squark masses and their implications on K and B physics have been extensively studied [5]. In the quark sector the Yukawa couplings define a pattern with a heavy third family and small mixing angles with the lighter families. At low energies, in the basis of quark mass eigenstates the squark mass matrix is not diagonal, with off-diagonal terms determined by this pattern. In particular, the top-quark corrections on the light quark sector are reduced by the small size of the mixings.[1] Here we would like to study the leptonic sector. Although charged-lepton and down-quark masses do not look too different, the results on neutrino oscillations (see [7] for a recent review) suggest a completely different pattern of Yukawa couplings. First of all, they have to accommodate large lepton mixings (3σ limits from [8]):

$$
\begin{aligned}
0.47 &< \sin\theta_{12} < 0.67 , \\
0.56 &< \sin\theta_{23} < 0.83 , \\
&\sin\theta_{13} < 0.23 .
\end{aligned}
\tag{1}
$$

Second, the observed mass differences [8]

$$5.4 < \Delta m^2_{21}/10^{-5}\,\mathrm{eV}^2 < 10$$

or

$$14 < \Delta m^2_{21}/10^{-5}\,\mathrm{eV}^2 < 19 ,$$

[1] See [6] for a recent analysis of the correlation between quark and lepton flavor changing processes in SUSY-GUTs. There it is shown that the strongest bounds on these models will be set by upcoming experiments on lepton decays.

$$1.5 \; < \; \Delta m^2_{32}/10^{-3}\,\mathrm{eV}^2 \; < \; 3.9 \tag{2}$$

can be realized in different ways: with hierarchical Yukawa couplings (mass differences of the order of the larger mass), or with almost degenerate couplings (mass differences much smaller than masses). In addition, the couplings can be large or small, since the values of the neutrino masses depend on the large masses M of the *right-handed* neutrinos that appear in the *seesaw* mechanism [9]. In this way, a mass of order 1 eV can be generated by a coupling $Y_\nu \approx 1$ with $M \approx 10^{14}$ GeV or by $Y_\nu \approx 10^{-2}$ with $M \approx 10^{10}$ GeV. Another difference with the quark sector has to do with the presence of more complex phases. In the neutrino sector all the low energy observables (three neutrino masses and three complex mixings) together with the masses of the three right-handed neutrinos do not fix completely the matrix $\mathbf{Y}_\nu$ of Yukawa couplings. There appears a complex orthogonal matrix $\mathbf{R}$ [10] that is important in the physics above the large scale M (for example, a complex $\mathbf{R}$ would be necessary to generate leptogenesis [11]).

In this paper we study the lepton flavor violating decays $\ell \to \ell'\gamma$ in SUSY models with universal scalar masses at the Planck scale. A first objective is to evaluate the relevance of the corrections due to the running of the masses between $M_\mathrm{P} = M_\mathrm{Planck}/\sqrt{8\pi} \approx 2 \times 10^{18}$ GeV and the GUT scale $M_X \approx 2 \times 10^{16}$. In particular, we will compare the three branching ratios $\ell \to \ell'\gamma$ taking universal masses at M_X and at M_P. We will include the case where the scalar masses vanish at M_P and are generated by (flavor blind) gaugino radiative corrections (the *no-scale* model). We will discuss three types of RG corrections introducing lepton flavor violation: corrections from the top quark Yukawa coupling, which affect the masses of the charged slepton singlets in SU(5) models; corrections from a large tau Yukawa coupling, which is the case in models with large $\tan\beta$, and corrections from the neutrino Yukawa couplings, which dominate in the usual models with universal masses at M_X. We show that the second type of corrections (usually neglected in the literature) can change the off-diagonal slepton masses in a $\approx 10\%$ for universal masses at M_X or by a factor of order 2 for universal masses at M_P.

A second objective in this paper is to analyze what the rate of lepton flavor violation is in the particular model of neutrino masses and mixings recently proposed by Mohapatra et al. [12]. They observe that the angles required in these experiments can be obtained from leptonic mixing angles identical to the ones in the quark sector at large scales (see Fig. 1). Although this possibility requires $\tan\beta \approx 50$ and some amount of fine tuning (the degeneracy of the neutrino masses must increase with the running, their values get *focused* in the infrared by tau corrections), we think it provides a well motivated scenario. It suggests that the dominant source of fermion mixing is in the interactions of down quarks and charged leptons, something natural in the simplest unification models (their couplings may unify, for example, in SU(5)). The large values of the tau Yukawa coupling required imply a violation of flavor symmetry that could be in conflict with the data.

In our analysis we will consider quasi-degenerate neutrinos, with a mass $m_\nu \approx 0.2$ eV that is compatible with

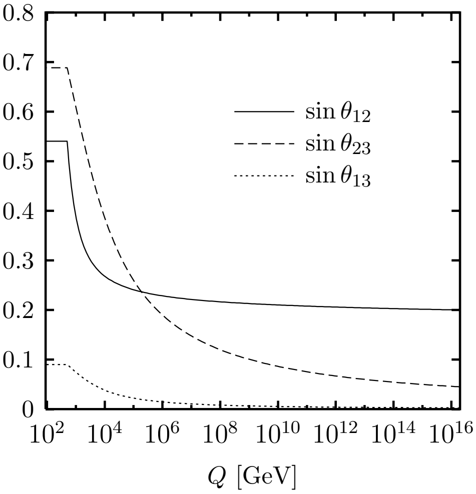

Fig. 1. Evolution of lepton mixing angles from CKM-like mixings at the GUT scale. The resulting low-energy angles are compatible with neutrino oscillation data ($\tan\beta = 50$)

$m_\nu < 2.2$ eV (^{3}H β-decay), $m_\nu < 0.7/3$ eV (MAW) and $m_\nu < (0.35$–$1.05)$ eV ($\beta\beta_{0\nu}$) [13]. We will assume that CP is conserved, with all the light neutrinos having the same CP parity and with a real matrix of Yukawa couplings. This implies a real matrix $\mathbf{R}$ and real neutrino mixings (it has been shown that a complex $\mathbf{R}$ may enhance the rate of lepton flavor violation by several orders of magnitude [14]).

For the mixing angles we will reproduce the values in (1), with θ_{13} below the bounds provided by the CHOOZ [15] and Palo Verde [16] experiments. One should note, however, that very small values of this angle are not stable under radiative corrections [17]. Taking a zero value at M_X we obtain at low energies values that go from $\theta_{13} \approx 0.1$ for large $\tan\beta$ to $\theta_{13} \approx 5 \times 10^{-5}$ for small $\tan\beta$ and low M (i.e. small values of all the lepton Yukawa couplings).

2 Radiative corrections to slepton masses

Universal SUSY-breaking masses generated by gravitational interactions receive radiative corrections from Yukawa interactions. In the minimal supersymmetric standard model (MSSM) with right-handed neutrinos the relevant trilinear couplings in the superpotential are[2]

$$\mathcal{W} = {Y_e}^{ij}\; E^c_i H_1 L_j + {Y_\nu}^{ij}\; N^c_i H_2 L_j \;, \tag{3}$$

where L_i, E^c_i and N^c_i stand for the three families of lepton doublets, charged singlets, and neutrino singlets, respectively. H_1 and H_2 are the MSSM Higgs-doublet superfields. We will assume that the neutrino singlets get a common mass at an intermediate scale M. Notice that the strength of the couplings necessary to reproduce the light neutrino mass spectrum depends on M, becoming top-quark-like when $M \approx 10^{14}$ GeV.[3]

[2] The contraction of SU(2) indices of two doublets is $AB \equiv \epsilon_{ab}A^aB^b$, where ϵ_{ab} is the antisymmetric tensor.

[3] The seesaw implies $Y_\nu \sim \sqrt{m_\nu M}/(v\sin\beta)$.

(i) Universal scalar masses at M_X. Most previous analyses of the RG corrections [10, 18–21] take universal slepton masses m_0 at the GUT scale M_X, neglecting their running between M_P and M_X. We will compare the results in that case with the ones in a minimal scenario with SU(5) gauge symmetry between both scales, and will show that the approximation (even in the no-scale case) is very poor.

We take at the GUT scale diagonal charged lepton couplings $\mathbf{Y}_e$ and include all the lepton mixing in $\mathbf{Y}_\nu$. Taking universal SUSY breaking masses[4] at M_X, the RG corrections introduce two relevant effects (see e.g. [10, 22] for the RG equations of the MSSM with right-handed neutrinos). First, the running down to M generates off-diagonal terms in the slepton-doublet mass matrix:

$$m^2_{\tilde{L}\,ij} \approx -\frac{3}{8\pi^2} m_0^2 (\mathbf{Y}_\nu^\dagger Y_\nu)_{ij} \log \frac{M_X}{M} \,. \tag{4}$$

Second, the running also generates off-diagonal terms in $\mathbf{Y}_e$. The low-energy charged-lepton mass matrix must be rediagonalized with rotations θ^e_{12}, θ^e_{13} and θ^e_{23} of order

$$\theta^e_{ij} \approx -\frac{1}{16\pi^2} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log \frac{M_X}{M} \tag{5}$$

in the space of the three lepton doublets. For large values of $\tan\beta$ the tau coupling separates the third slepton family from the other two (it decreases $m^2_{\tilde{L}\,33}$ by some 20%; see below). When the rotation (5) is performed also in the space of slepton doublets, it induces new off-diagonal terms. More precisely, it gives a correction (usually neglected in the literature) of order $1/(16\pi^2) Y_\tau^2 \log(M_X/M_Z)$ to the terms $m^2_{\tilde{L}\,i3}$ in (4).

To quantify the lepton–slepton misalignment we show $\mathbf{m}^2_{\tilde{\mathbf{L}}}$ at the electroweak scale M_Z for $m_0 = 300$ GeV at M_X. We set the common gaugino mass $m_{1/2} = 300$ GeV and distinguish large and small values of $\tan\beta$ (50 and 3, respectively). We take in both cases large values of $\mathbf{Y}_\nu$ (around 0.9 at M_X, corresponding to $M = 10^{14}$ GeV) that reproduce the observed pattern of neutrino mixings and mass differences with $m_i \approx 0.2$ eV, obtaining

$$\mathbf{m}^2_{\tilde{L}} = (353\,\mathrm{GeV})^2 \times \begin{pmatrix} 1 & -10^{-4} & -2\times10^{-5} \\ -10^{-4} & 0.999 & -5\times10^{-4} \\ -2\times10^{-5} & -5\times10^{-4} & 0.793 \end{pmatrix} \quad (\tan\beta = 50)\,, \tag{6}$$

$$\mathbf{m}^2_{\tilde{L}} = (352\,\mathrm{GeV})^2 \times \begin{pmatrix} 1 & -5\times10^{-5} & 5\times10^{-5} \\ -5\times10^{-5} & 0.997 & -3\times10^{-3} \\ 5\times10^{-5} & -3\times10^{-3} & 0.996 \end{pmatrix} \quad (\tan\beta = 3)\,. \tag{7}$$

The differences in the off-diagonal terms are due to the different mixings assumed at the GUT scale (CKM-like for $\tan\beta = 50$ and bimaximal for $\tan\beta = 3$), with a 10% contribution to $m^2_{\tilde{L}\,i3}$ from the rediagonalization of the charged lepton mass matrix in the case of large $\tan\beta$. The differences in the diagonal terms are only due to tau corrections (negligible for $\tan\beta = 3$).

(ii) Universal scalar masses at M_P. Now let us give an estimate of the running that includes RG corrections between M_P and M_X. We will consider an SU(5) grand unification framework [4, 23, 24], as this is the simplest possibility. The LFV effects that we will find are minimal in the sense that the unified theory could contain other sizable family-dependent couplings in addition to the ones required to embed the MSSM. We do not include threshold effects at the GUT scale [4], which could be as well an important source of lepton flavor asymmetry [25].

In SUSY-SU(5) each generation of quark doublets, up-quark singlets and charged-lepton singlets can be accommodated in the same **10** irrep (Ψ_i) of the group, whereas lepton doublets and down-quark singlets would be in the $\overline{\mathbf{5}}$ (Φ_i). We also need gauge singlets (N_i^c) to generate neutrino masses, and a vector-like $\mathbf{5}+\overline{\mathbf{5}}$ (H_2 and H_1) to include the two standard Higgs doublets. Other vector-like fermions or the Higgs representations needed to break the GUT symmetry are not essential in our calculation. Including just the three fermion families the trilinear terms in the superpotential read[5] [26]

$$\mathcal{W}_{\mathrm{SU}(5)} = \frac{1}{4} Y_u^{ij}\, \Psi_i \Psi_j H_2 + \sqrt{2}\, Y_{d/e}^{ij}\, \Psi_i \Phi_j H_1 + Y_\nu^{ij}\, N_i^c \Phi_j H_2 \,. \tag{8}$$

At M_X we do the matching of Yukawa couplings in the following way.

(i) First we diagonalize $\mathbf{Y}_d$ and $\mathbf{Y}_e$, and include all the quark mixing in $\mathbf{Y}_u$ and all the lepton mixing in $\mathbf{Y}_\nu$.

(ii) The matrix $\mathbf{Y}_u$ (symmetrized through a rotation of the up quark singlets) is then matched to the analogous matrix above M_X.

(iii) We do *not* assume tau–bottom unification. To define $\mathbf{Y}_{\mathbf{d/e}}$ we determine the tau and the bottom masses at M_X (m_τ^0 and m_b^0); if the tau mass is larger, as it is usually the case for both the small and the large values of $\tan\beta$ that we consider, we match $\mathbf{Y}_{\mathbf{d/e}}$ to $\mathbf{Y}_\mathbf{e}$. The smaller bottom mass would then be explained through mixing (θ) of the bottom component in Φ_3 with the bottom component in a $\Phi + \overline{\Phi}$, which would reduce its coupling by a factor of $\cos\theta = m_b^0/m_\tau^0$. For intermediate values of $\tan\beta$ the bottom is heavier than the tau at M_X, so we would proceed in the opposite way. The lighter charged-lepton and down-quark masses could be separated by higher dimensional operators, but the radiative corrections that they introduce are irrelevant.

The matching of SUSY-breaking scalar masses is straightforward for squark doublets, up squark singlets, charged slepton singlets (all of them equal to $\mathbf{m}^2_{\tilde{\Psi}}$ between M_X and M_P) and sneutrino singlets ($\mathbf{m}^2_{\tilde{N}}$). For down squark singlets and slepton doublets we take into account the mixing with vector-like fields, which reduces the RG correc-

[4] We neglect the scalar trilinears.

[5] $\Psi\Psi H_2 \equiv \epsilon_{abcde}\ \Psi^{ab}\Psi^{cd}H_2^e$, $\Psi\Phi H_1 \equiv \Psi^{ab}\Phi_a H_{1b}$ and $N^c\Phi H_2 \equiv N^c\Phi_a H_2^a$, where $a,\ldots,e = 1,\ldots,5$ are SU(5) indices and ϵ_{abcde} is the totally antisymmetric tensor.

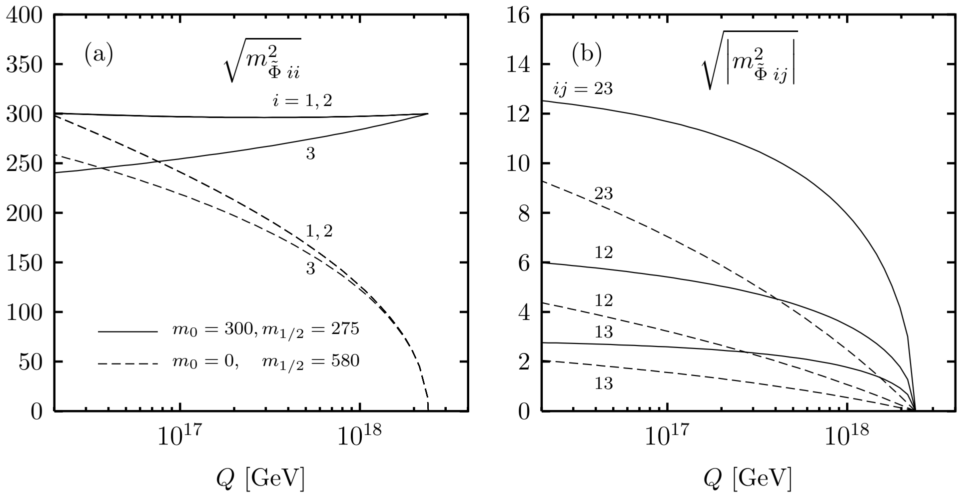

Fig. 2a,b. Mass splittings and off-diagonal terms [GeV] generated in the running of $\mathbf{m}^2_{\tilde\Phi}$ from M_P to M_X. The diagonal terms for the first and second families coincide. Dashed lines correspond to no-scale models ($m_0 = 0$ at M_P)

tions of the Yukawa couplings on the bottom squark mass squared by a factor of $\cos^2\theta$.

We give in the appendix the RG equations between M_X and M_P of Yukawa couplings and SUSY-breaking masses. The running introduces three main effects on the flavor structure of the model; two of them add to the corrections described for universal masses at M_X, whereas the third one is new. At M_X there appear new off-diagonal terms in the slepton-doublet mass matrix:

$$m^2_{\tilde\Phi\,ij} \approx -\frac{3}{8\pi^2} m_0^2 (\mathbf{Y}^\dagger_\nu \mathbf{Y}_\nu)_{ij} \log\frac{M_\mathrm{P}}{M_X}\ . \tag{9}$$

The second effect is a large mass separation of the third slepton family produced by tau corrections:

$$m^2_{\tilde\Phi\,33} - m^2_{\tilde\Phi\,ii} \approx -\frac{3}{2\pi^2} m_0^2\ Y_\tau^2\ \log\frac{M_\mathrm{P}}{M_X}\ . \tag{10}$$

This mass splitting will introduce off-diagonal mass terms once the charged-lepton Yukawa matrix is rediagonalized at low energies. The final (new) effect has to do with the slepton-singlet mass matrix. In the minimal SU(5) model this matrix coincides with the one for up squarks, so it will be affected by large top quark radiative corrections. At M_X there will be off-diagonal terms of order

$$m^2_{\tilde\Psi\,ij} \approx -\frac{9}{8\pi^2} m_0^2 (\mathbf{Y}^\dagger{}_u \mathbf{Y}_u)_{ij} \log\frac{M_\mathrm{P}}{M_X}\ , \tag{11}$$

where $\mathbf{Y}_u$ contains the whole CKM rotation (we take $\mathbf{Y}_{d/e}$ diagonal at M_X). This effect was first considered in [23].

To illustrate the relevance of the corrections between M_P and M_X we give $\mathbf{m}^2_{\tilde L}$ at M_Z and compare it with the matrices obtained for universal scalar masses at M_X. We take $m_{1/2} = 275\,\mathrm{GeV}$ and $m_0 = 300\,\mathrm{GeV}$ at M_P, which give at M_X values similar to the ones used in (6) and (7):

$$\mathbf{m}^2_{\tilde L} = (353\,\mathrm{GeV})^2 \times \begin{pmatrix} 1 & -4\times10^{-4} & -7\times10^{-5} \\ -4\times10^{-4} & 0.997 & -2\times10^{-3} \\ -7\times10^{-5} & -2\times10^{-3} & 0.567 \end{pmatrix}$$

$$(\tan\beta = 50)\ , \tag{12}$$

$$\mathbf{m}^2_{\tilde L} = (349\,\mathrm{GeV})^2 \times \begin{pmatrix} 1 & -2\times10^{-4} & 2\times10^{-4} \\ -2\times10^{-4} & 0.990 & -10^{-2} \\ 2\times10^{-4} & -10^{-2} & 0.989 \end{pmatrix}$$

$$(\tan\beta = 3)\ . \tag{13}$$

A final comment concerns the no-scale models. In these models all scalar SUSY-breaking masses are zero at the Planck scale and non-zero values are generated mainly by *flavor blind* gaugino corrections (see the RG equations in the appendix). Once the scalar masses are generated, however, family-dependent Yukawas will separate them. Therefore, it is not clear whether or not it is justified to take universal masses at M_X in this scenario. Contrary to the usual claim, we find that the mass splittings and the off-diagonal terms in the scalar sector at M_X are only reduced by a factor of ≈ 0.8 if the initial masses are taken zero at M_P. We show in Fig. 2a $\sqrt{m^2_{\tilde\Phi\,ii}}$ at M_X for $m_0 = 300\,\mathrm{GeV}$ and $m_{1/2} = 275\,\mathrm{GeV}$ and for $m_0 = 0\,\mathrm{GeV}$ and $m_{1/2} = 580\,\mathrm{GeV}$ at M_P, taking $M = 10^{14}\,\mathrm{GeV}$ and a large value of $\tan\beta$. In the first case we obtain $\sqrt{m^2_{\tilde\Phi\,11}} \approx \sqrt{m^2_{\tilde\Phi\,22}} = 300\,\mathrm{GeV}$, and $\sqrt{m^2_{\tilde\Phi\,33}} = 240\,\mathrm{GeV}$, whereas in the no-scale case we get $\sqrt{m^2_{\tilde\Phi\,11}} \approx \sqrt{m^2_{\tilde\Phi\,22}} = 300\,\mathrm{GeV}$, and $\sqrt{m^2_{\tilde\Phi\,33}} = 260\,\mathrm{GeV}$. In Fig. 2b we plot $\sqrt{|m^2_{\tilde\Psi\,i3}|}$ in the same two cases, with qualitatively the same behaviour for this parameter. We conclude that the no-scale possibility does *not* justify taking universal SUSY-breaking scalar masses at the GUT scale M_X.

3 Lepton flavor violation: $\mu\to e\gamma$, $\tau\to e\gamma$, $\tau\to\mu\gamma$

Let us now determine how the misalignment between lepton and slepton families translates into flavor violating decays.

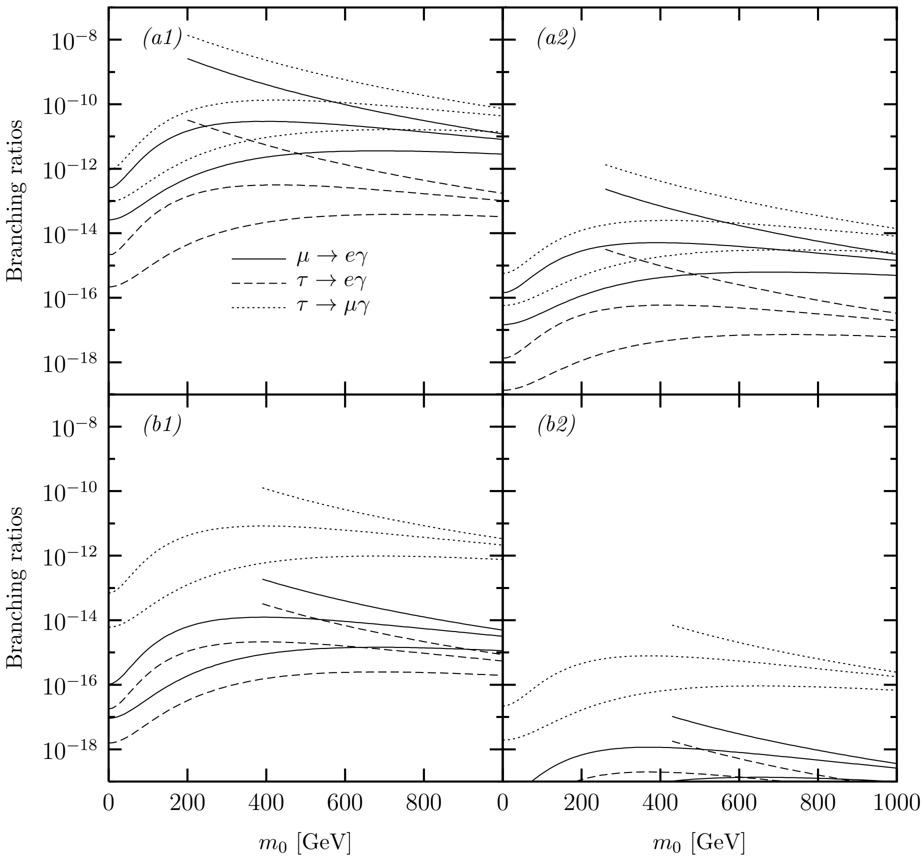

Fig. 3. The same as in Fig. 2 but assuming universality at $M_{\rm P}$. Branching ratios of $\ell \to \ell'\gamma$ for $m_{1/2} = 100, 300, 500$ GeV and different values of the scalar mass parameter m_0 at M_X. Cases **a** and **b** correspond to $\tan\beta = 50, 3$, whereas cases **1** and **2** correspond to $M = 10^{14}, 5\times10^{11}$ GeV, respectively. Lower values of m_0 for $m_{1/2} = 100$ GeV give slepton masses excluded by present bounds

The present experimental limits are

$$\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11} \quad [27] \,, \tag{14}$$

$$\mathrm{BR}(\tau \to e\gamma) < 2.7 \times 10^{-6} \quad [28] \,, \tag{15}$$

$$\mathrm{BR}(\tau \to \mu\gamma) < 6 \times 10^{-7} \quad [29] \,, \tag{16}$$

whereas future searches will be sensitive to branching ratios of up to

$$\mathrm{BR}(\mu \to e\gamma) < 10^{-14} \quad [30] \,, \tag{17}$$

$$\mathrm{BR}(\tau \to \mu\gamma) < 10^{-9} \quad [31] \,. \tag{18}$$

A detailed description of all the diagrammatics involved in these processes can be found in [32]. Here we will just apply that calculation to the particular set of parameters described in the previous section.

We will distinguish four cases: large (a) and small (b) values of $\tan\beta$ (50 and 3, respectively), and large (1) and small (2) values of the neutrino Yukawa couplings (corresponding to values of M of 10^{14} and 5×10^{11} GeV, respectively). As explained in the introduction, the large values of $\tan\beta$ define a consistent scenario with unification of the quark and lepton mixing angles at M_X. In the cases with $\tan\beta = 3$ we take a bimaximal mixing with $\theta_{13} = 0$ at M_X, which corresponds to low-energy values $\theta_{13} \approx 10^{-4}$. Values of θ_{13} up to 10^{-2} give negligible effects on the branching ratios, whereas larger values (up to the experimental limit in (1)) give important corrections to the rates of $\mu \to e\gamma$ and $\tau \to e\gamma$.

(i) Universal scalar masses at M_X. In our numerical analysis we take a universal gaugino mass $m_{1/2} = 100, 300, 500$ GeV at the GUT scale. In each case we vary the common slepton mass parameter m_0 at M_X between 1 TeV and the minimum value that gives acceptable masses for all the slepton fields at low energies (we take the bounds from [33]). Notice that this minimum value strongly depends on the gaugino mass parameter, as it gives important RG corrections that increase the low-energy value of the slepton masses.

The results for the four cases (a1, a2, b1, b2) are summarized in Fig. 3. For each case we plot the branching ratios of $\mu \to e\gamma$ (solid), $\tau \to e\gamma$ (dashes) and $\tau \to \mu\gamma$ (dots). In all the cases the three branching ratios are dominated by diagrams with exchange of charginos and sneutrinos [32]. The rates of $\mu \to e\gamma$ and $\tau \to e\gamma$ in the cases with low $\tan\beta$ would be scaled by a factor $\approx (\theta_{13}/10^{-2})^2$ for values of this angle close to the experimental bound $\theta_{13} \approx 0.2$.

(ii) Universal scalar masses at $M_{\rm P}$

Now we take $m_{1/2} = 100, 300, 500$ GeV at the Planck scale and universal scalar masses $\mathbf{m}^2_{\tilde\Phi} = \mathbf{m}^2_{\tilde\Psi} = m_0^2\mathbf{1}$ also at the same scale. Again, we vary the parameter m_0 between 1 TeV and the minimum value not excluded experimentally. The value $m_0 = 0$ at $M_{\rm P}$, corresponding to no-scale models, is acceptable for large gaugino masses.

The results for the four cases described above are given in Fig. 4. The process $\mu \to e\gamma$ is dominated by chargino–sneutrino diagrams for large neutrino Yukawa couplings (cases a1 and b1) and by neutralino–slepton singlet diagrams otherwise. These second diagrams also dominate in

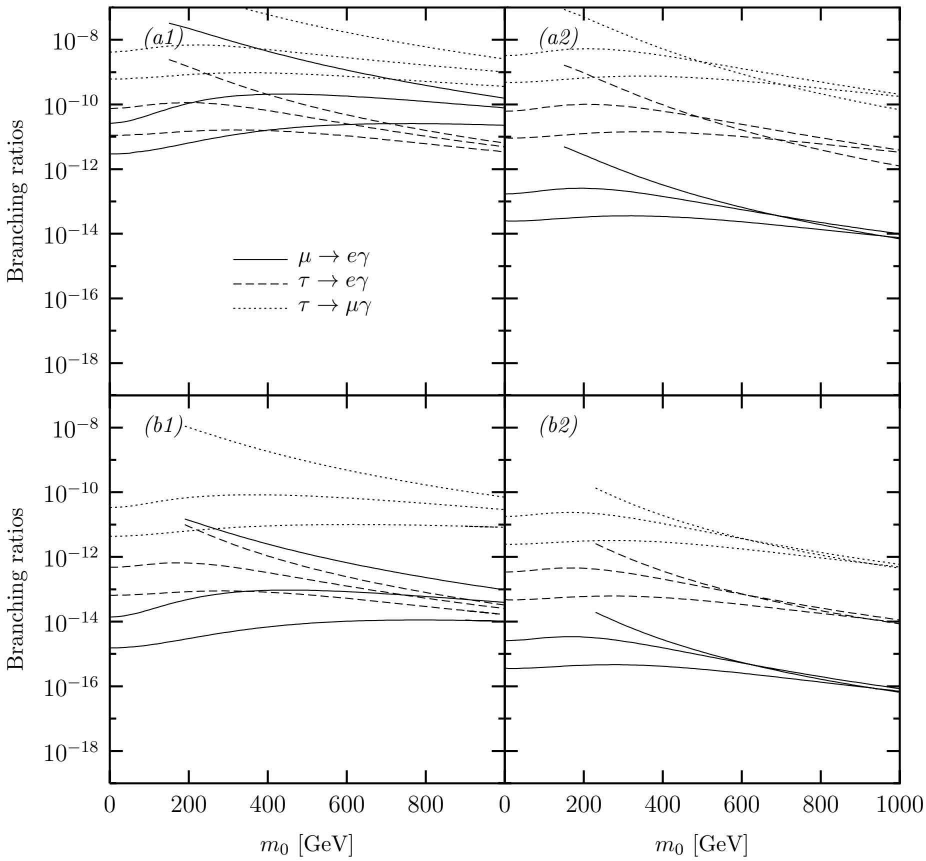

Fig. 4. Branching ratios of $\ell \to \ell'\gamma$ for $m_{1/2} = 100, 300, 500\,\mathrm{GeV}$ and different values of the scalar mass parameter m_0 at M_P. Cases **a** and **b** correspond to $\tan\beta = 50, 3$, whereas cases **1** and **2** correspond to $M = 10^{14}, 5\times 10^{11}$ GeV, respectively. $m_0 = 0$ defines a no-scale scenario

$\tau \to \mu\gamma$ and $\tau \to e\gamma$ for all the values of $\tan\beta$ and M analyzed. The rate of $\mu \to e\gamma$ in the case b1 would scale by a factor of $\approx (\theta_{13}/10^{-2})^2$ for larger values of this mixing angle, whereas the rest of the processes in case b1 and the three processes in case b2 would not change notably.

4 Discussion

The first question that we would like to address is how relevant are the RG corrections between the Planck and the GUT scales. Comparing Figs. 3 and 4 we find, for example, that these corrections can increase $\mathrm{BR}(\tau \to \mu\gamma)$ in four orders of magnitude (case a2) or $\mathrm{BR}(\mu \to e\gamma)$ in two orders of magnitude (case b2). The approximation of equal slepton masses at M_X is then clearly not justified. As discussed in Sect. 2, even in no-scale models it gives a poor estimate of the slepton mass matrix at low energies.

We have shown that in the running between M_P and M_X both top quark and tau lepton corrections may be relevant. Top corrections affect the charged slepton singlet masses $m^2_{\tilde{E}^c\,ij}$, making the contributions mediated by slepton singlet and neutralino of the same order as the ones mediated by slepton doublet and chargino (which are proportional to $m^2_{\tilde{L}\,ij}$). These corrections have been analyzed in the past [23,34] in some detail. On the other hand, large tau corrections (in the large $\tan\beta$ regime) decrease the mass of the third slepton family, which introduces off-diagonal terms when the slepton matrix is rotated to the basis of charged-lepton mass eigenstates (see (5) and (10); the charged-lepton Yukawas get off-diagonal terms in the running down to low energies). The analysis of tau corrections that we present here is absent in all previous studies of $\ell \to \ell'\gamma$, and it is relevant since it may amplify by a factor of 2 the rate of lepton flavor violation (notice that these corrections add to the linear scaling of the amplitudes with $\tan^2\beta$ [35]).

A second point in our work is to establish if the large $\tan\beta$ model proposed by Mohapatra et al. in [12] respects the present bounds on $\ell \to \ell'\gamma$, and what would be the prospects at future experiments. From cases (a1) and (a2) in Fig. 4 we see that large values of the neutrino couplings ($M = 10^{14}\,\mathrm{GeV}$) would imply a $\mathrm{BR}(\mu \to e\gamma)$ already excluded. On the other hand, low values of the neutrino couplings ($M = 5 \times 10^{11}\,\mathrm{GeV}$) would make the model consistent with all data, although the non-observation of $\mu \to e\gamma$ at PSI [30] or of $\tau \to \mu\gamma$ at KEK [31] would exclude this model. These conclusions would be completely different if one neglects the running between M_P and M_X (see Fig. 3), as the model could be consistent with the non observation of lepton flavor violation at present and near future experiments.

From Fig. 4 we can extract as well how severe is the generic flavor problem of SUSY models in the lepton sector. For low values of $\tan\beta$ the tau coupling is small, and we obtain (for $\theta_{13} < 0.01$) $\mathrm{BR}(\mu \to e\gamma) \lesssim 10^{-11}$ (see the cases b1 and b2 in Fig. 4), a branching ratio that is within the present experimental limits. Larger values of $\tan\beta$ will always put constraints on the SUSY-breaking mass parameters. For example, the large values of $\tan\beta$ required in [12] respect the bounds on $\mu \to e\gamma$ only if the gaugino mass parameter $m_{1/2}$ is larger than 500 GeV (case a1 in Fig. 4) or if the neutrino Yukawa couplings are small enough, with $M \lesssim 10^{11}$ GeV (case a2 in Fig. 4).

A final point is whether these SUGRA models imply that lepton flavor violating processes are necessarily going to be observed in upcoming experiments. Again, both tau and neutrino Yukawa couplings play a relevant role in the answer to that question. If all the lepton couplings are small (i.e. for low values of $\tan\beta$ and M), taking $m_{1/2}, m_0 \gtrsim$ 500 GeV we find $\mathrm{BR}(\mu \to e\gamma) \lesssim 10^{-15}$, a value that could avoid the 10^{-14} limit projected at PSI [30]. For lighter SUSY-breaking masses or larger values of $\tan\beta$ or M the gravity-mediated scenario of SUSY-breaking that we have considered predicts accessible rates of $\mu \to e\gamma$. Another process with good experimental prospects is $\tau \to \mu\gamma$. This process is sensitive to the tau coupling and less dependent on neutrino couplings (see the different cases in Fig. 4). For $m_{1/2} = 300$ GeV its branching ratio goes from 10^{-8} for large $\tan\beta$ to 10^{-11} for low values of $\tan\beta$ and M. In summary, the non-observation of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ would imply very low values of the Yukawas of the neutrinos *and* of $\tan\beta$. Comparing with the plots in Fig. 3, we see that this generic conclusion cannot be obtained if one assumes universality of the SUSY-breaking masses at M_{GUT} instead of M_P.

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5 RG corrections between M_{P} and M_X

The relevant RG equations between the Planck and the GUT scales are given below [24]. We neglect the scalar trilinears and define $t \equiv \log(Q/Q_0)$.
(1) Gauge coupling [$\alpha_G = g_5^2/(4\pi)$]:

$$\frac{\mathrm{d}\alpha_G}{\mathrm{d}t} = -\frac{3}{2\pi}\alpha_G^2 \ . \tag{19}$$

(2) Gaugino mass:

$$\frac{\mathrm{d}M_5}{\mathrm{d}t} = -\frac{3}{2\pi}\alpha_G M_5 \ . \tag{20}$$

(3) Yukawa couplings:

$$\begin{aligned} 16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{Y}_u &= \mathbf{Y}_u \left[3\mathrm{Tr}(\mathbf{Y}_u^\dagger \mathbf{Y}_u) + \mathrm{Tr}(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) + 2\mathbf{Y}_{d/e}^\dagger \mathbf{Y}_{d/e} + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u\right] \\ &\quad + \left(2\mathbf{Y}_{d/e}^\dagger \mathbf{Y}_{d/e} + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u\right)^{\mathrm{T}} \mathbf{Y}_u - \frac{96}{5} g_5^2 \mathbf{Y}_u \ , \end{aligned} \tag{21}$$

$$\begin{aligned} 16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{Y}_{d/e} &= \mathbf{Y}_{d/e}\left[4\mathrm{Tr}(\mathbf{Y}_{d/e}^\dagger \mathbf{Y}_{d/e}) + 2\mathbf{Y}_{d/e}^\dagger \mathbf{Y}_{d/e} + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u\right] \\ &\quad + \left[2\mathbf{Y}_{d/e}\mathbf{Y}_{d/e}^\dagger + 3\left(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right)^{\mathrm{T}}\right]\mathbf{Y}_{d/e} - \frac{84}{5} g_5^2 \mathbf{Y}_{d/e} \ , \end{aligned} \tag{22}$$

$$\begin{aligned} 16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{Y}_\nu &= \mathbf{Y}_\nu \left[3\mathrm{Tr}(\mathbf{Y}_u^\dagger \mathbf{Y}_u) + \mathrm{Tr}(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) + 4\left(\mathbf{Y}_{d/e}\mathbf{Y}_{d/e}^\dagger\right)^{\mathrm{T}}\right] \\ &\quad + 5\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu - \frac{48}{5} g_5^2 \mathbf{Y}_\nu \ . \end{aligned} \tag{23}$$

(4) SUSY-breaking masses:

$$\begin{aligned} 8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{m}_{\tilde\Psi}^2 &= \mathbf{m}_{\tilde\Psi}^2 \left(\mathbf{Y}_{d/e}^\dagger \mathbf{Y}_{d/e} + \frac{3}{2}\mathbf{Y}_u^\dagger \mathbf{Y}_u\right) \\ &\quad + \left(\mathbf{Y}_{d/e}^\dagger \mathbf{Y}_{d/e} + \frac{3}{2}\mathbf{Y}_u^\dagger \mathbf{Y}_u\right)\mathbf{m}_{\tilde\Psi}^2 \\ &\quad + 2\mathbf{Y}_{d/e}^\dagger \left(\mathbf{m}_{\tilde\Phi}^{2\mathrm{T}} + m_{H_1}^2\right)\mathbf{Y}_{d/e} + 3\mathbf{Y}_u^\dagger \left(\mathbf{m}_{\tilde\Psi}^{2\mathrm{T}} + m_{H_2}^2\right)\mathbf{Y}_u \\ &\quad - \frac{72}{5} g_5^2 M_5^2 \ , \end{aligned} \tag{24}$$

$$\begin{aligned} 8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{m}_{\tilde\Phi}^2 &= \mathbf{m}_{\tilde\Phi}^2 \left[2(\mathbf{Y}_{d/e}\mathbf{Y}_{d/e}^\dagger)^{\mathrm{T}} + \frac{1}{2}\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right] \\ &\quad + \left[2(\mathbf{Y}_{d/e}\mathbf{Y}_{d/e}^\dagger)^{\mathrm{T}} + \frac{1}{2}\mathbf{Y}_\nu {\mathbf{Y}_\nu}^\dagger\right]\mathbf{m}_{\tilde\Phi}^2 \\ &\quad + 4\mathbf{Y}_{\mathbf{d/e}}^*(\mathbf{m}_{\tilde\Psi}^{2\mathrm{T}} + m_{H_1}^2)\mathbf{Y}_{d/e}^\dagger + \mathbf{Y}_\nu^\dagger(\mathbf{m}_{\tilde N}^{2\mathrm{T}} + m_{H_2}^2)\mathbf{Y}_\nu \\ &\quad - \frac{48}{5} g_5^2 M_5^2 \ , \end{aligned} \tag{25}$$

$$\begin{aligned} 8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{m}_{\tilde N}^2 &= \frac{5}{2}\mathbf{m}_{\tilde N}^2(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)^{\mathrm{T}} + \frac{5}{2}(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)^{\mathrm{T}}\mathbf{m}_{\tilde N}^2 \\ &\quad + 5\mathbf{Y}_\nu^*(\mathbf{m}_{\tilde\Phi}^{2\mathrm{T}} + m_{H_2}^2)\mathbf{Y}_\nu^\dagger \ , \end{aligned} \tag{26}$$

$$\begin{aligned} 8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} m_{H_1}^2 &= 4m_{H_1}^2 \mathrm{Tr}(\mathbf{Y}_{d/e}^\dagger \mathbf{Y}_{d/e}) + 4\mathrm{Tr}(\mathbf{Y}_{d/e}\mathbf{m}_{\tilde\Psi}^2 \mathbf{Y}_{d/e}^\dagger) \\ &\quad + 4\mathrm{Tr}(\mathbf{Y}_{d/e}^\dagger \mathbf{m}_{\tilde\Phi}^{2\mathrm{T}} \mathbf{Y}_{d/e}) - \frac{48}{5} M_5^2 \ , \end{aligned} \tag{27}$$

$$\begin{aligned} 8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} m_{H_2}^2 &= m_{H_2}^2 \left[3\mathrm{Tr}(\mathbf{Y}_u^\dagger \mathbf{Y}_u) + \mathrm{Tr}(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)\right] + 6\mathrm{Tr}(\mathbf{Y}_u^\dagger \mathbf{m}_{\tilde\Psi}^{2\mathrm{T}} \mathbf{Y}_u) \\ &\quad + \mathrm{Tr}(\mathbf{Y}_\nu^\dagger \mathbf{m}_{\tilde N}^{2\mathrm{T}} \mathbf{Y}_\nu) + \mathrm{Tr}(\mathbf{Y}_\nu \mathbf{m}_{\tilde\Phi}^2 \mathbf{Y}_\nu^\dagger) - \frac{48}{5} M_5^2 \ . \end{aligned} \tag{28}$$

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